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Some Rayleigh-Taylor Stabilizing Mechanisms in Thin Laser Targets

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Plasma Physics Division

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Several mechanisms that reduce the linear growth rate and severity of Rayleigh-Taylor instability in thin, inhomogeneous laser targets are discussed.

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CONTENTS

SOME RAYLEIGH-TAYLOR STABILIZING MECHANISMS IN THIN LASER TARGETS

I. INTRODUCTION

The Rayleigh-Taylor[1] instability arises in the acceleration of a fluid by one of lower density. This instability represents a potential obstacle to achieving laser-induced fusion in that it might destroy the symmetry of an imploding fuel pellet.

There are several surfaces in a laser-driven plasma at which the growth of Rayleigh-Taylor instability can be large. One such surface is the ablation layer separating the cold, high density fluid from the hot, low density material ablated from the surface. In an inhomogeneous target an instability occurs at any fluid interface where the acceleration is directed towards the denser material.

Several stabilizing mechanisms for the Rayleigh-Taylor instability are well known. In this paper we consider several mechanisms that tend to reduce the instability in thin, inhomogeneous laser targets. Some of the mechanisms we discuss have been treated in the literature. Our reason for reviewing them here is the increased relevance of thin-shell pellet designs in laser-fusion schemes.

Inhomogeneous pellets have been considered for use in laser-fusion schemes for some time. However, recent claims from the Lebedev Physical Institute[2] suggest that certain inhomogeneous hollow pellets with very thin shells having a ratio of radius R to thickness ΔR of as much as 100 can be imploded to yield an energy gain of as much as 10^3. The number of e-foldings during time t of a perturbation of

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wavenumber k expected in a homogeneous, incompressible, inviscid material having acceleration g against vacuum is $(kgt^2)^{\frac{1}{2}}$ in the standard, linear Rayleigh-Taylor problem.[3] The mode most likely to break up an imploding shell has wavenumber $k \sim 1/\Delta R$. Then if the time of implosion is given by $R \sim gt^2/2$, the number of e-foldings is of order $(2R/\Delta R)^{\frac{1}{2}}$. This crude analysis indicates that shells having an aspect ratio $R/\Delta R \leq 100$ should be highly susceptible to Rayleigh-Taylor instability. Claims to the contrary warrant further consideration.

The stabilizing mechanisms discussed here reduce the growth rate of the Rayleigh-Taylor instability in thin shells but probably cannot by themselves account for stability of shells having aspect ratio of 100.

II. Finite Layer Effects

In seeking effects that tend to stabilize imploding thin shells against the Rayleigh-Taylor instability, we consider first the simplest case, and find a rather large effect. We examine the effect of the finite widths of the fluids on the growth rate of perturbations at the fluid interface. The simplest case is represented by a model consisting of two slabs of inviscid, incompressible fluid of finite widths, d_1, d_2, and uniform densities, ρ_1 and ρ_2.

The linearized two-dimensional fluid equations for inviscid, incompressible fluids at rest are[3]

$$\frac{d}{dx}\delta u + ik\delta v = 0 \qquad \text{(divergence-free velocity)}$$

$$\gamma\delta\rho + \left(\frac{d\rho}{dx}\right)\delta u = 0 \qquad \text{(conservation of mass)} \tag{1}$$

$$\gamma\rho\,\delta u = -\frac{d}{dx}\delta P + g\,d\rho \qquad \text{(conservation of x-momentum)}$$

$$\gamma\rho\,\delta v = -ik\delta P \qquad \text{(conservation of y-momentum)}$$

Here $\hat{x}\delta u + \hat{y}\delta v$ is the velocity perturbation, δP the pressure perturbation and $\delta\rho$ the density perturbation of a fluid element of density ρ, subjected to an effective gravity $g\hat{x}$. The perturbations have been decomposed by a Fourier transform into normal modes, whose dependence on y and t is given by

$$\exp(\gamma t + iky),$$

where γ is the constant growth rate, and k is the constant wavenumber of a Rayleigh-Taylor mode.

The equations (1) reduce to

$$\frac{d}{dx}\left(\rho\frac{d}{dx}\delta u\right) - \rho k^2\delta u = \frac{k^2}{\gamma^2} g \left(\frac{d\rho}{dx}\right)\delta u \tag{2}$$

At a fluid interface the normal component of velocity is continuous, as is the quantity found by integrating (2) across the interface; the jump conditions at a fluid interface are therefore

$$\Delta[\delta u] = 0 \tag{3a}$$

$$\Delta\left[\rho\left(\frac{d}{dx}\delta u - \frac{k^2 g}{\gamma^2}\delta u\right)\right] = 0 \tag{3b}$$

If the two incompressible slabs of uniform density are confined between rigid planes at $x = -d_1$ and $x = d_2$, then the boundary conditions there are $\delta u = 0$. These boundary conditions, together with the jump conditions (3) at the interface and the differential equation (2), which reduces to $d^2\delta u/dx^2 - k^2\delta u = 0$, imply a growth rate given by[4]

$$\frac{\gamma^2}{gk} = \frac{\rho_1 - \rho_2}{\rho_1 \operatorname{ctnh} kd_1 + \rho_2 \operatorname{ctnh} kd_2} \tag{4}$$

If instead of being confined between rigid planes, the surfaces are free[5], then the boundary condition $d\delta P/dt = 0$ pertains. This condition leads to boundary conditions identical to the jump condition (3b), namely

$$\frac{d}{dx}\,\delta u - \frac{k^2 g}{\gamma^2}\,\delta u = 0 \qquad \text{(free boundary)} \qquad (5)$$

With the free boundary conditions, the growth rate of Rayleigh-Taylor modes is

$$\frac{\gamma^2}{gk} = \frac{\rho_1 - \rho_2}{\rho_1 \operatorname{ctnh} kd_2 + \rho_2 \operatorname{ctnh} kd_1} \qquad (6)$$

The growth rates in (4) and (6) are both less than the corresponding rates for semi-infinite fluids, and can be significantly less for perturbation wavelengths long compared to the widths of the slabs.

In the limit of small kd, the growth rate given by (6) approaches zero as $\gamma \to k\left[\, g d_1 d_2 (\rho_1 - \rho_2)/(\rho_1 d_1 + \rho_2 d_2)\right]^{\frac{1}{2}}$. The growth rate of (4) also scales as k in the long-wavelength limit. If for example, kd_1, kd_2, and ρ_2/ρ_1 are .1, then γ decreases to 30% of the thick shell value.

The growth rates found for this planar geometry may be applied to an imploding spherical shell as well by the presciption $k \to (\ell + \frac{1}{2})/R$, where R is the radius and ℓ is the degree of a spherical harmonic in a decomposition of the Rayleigh-Taylor perturbation. The correspondence is valid until spherical convergence effects near the origin can no longer be neglected, that is, until $(dR/dt)^2 \ll R\, d^2R/dt^2$ is no longer satisfied.[6]

III. Thin, High-Density Barrier

Some pellet designs call for a thin layer of high-density material to be sandwiched between the deuterium-tritium fuel shells on the inside and a moderate-density ablator on the outside. In this section, as in the preceding, we use the simplest model that provides a qualitative understanding of the effect of interest. In this case, we seek the effect of an incompressible layer of uniform density ρ_2 and width δ sandwiched between semi-infinite incompressible fluids of uniform densities ρ_1 and ρ_3 on either side. With the boundary conditions of vanishing perturbation at infinity, and the jump conditions (3) applied at the two interfaces, the growth rate is given implicitly by

$$(\Gamma + \frac{\rho_3}{\rho_2})(\frac{1}{\Gamma} + \frac{\rho_1}{\rho_2}) = e^{-2k\delta}\,(1 - \frac{\rho_3}{\rho_2})(1 - \frac{\rho_1}{\rho_2}) \tag{7}$$

in which $\Gamma \equiv (\nu^2 - kg)/(\nu^2 + kg)$.

We wish to consider the limit of (7), in which the intermediate layer is thin, $\delta \ll 1/k$, and of high density, $\rho_2 \gg \rho_1$ and $\rho_2 \gg \rho_3$, but in which the total mass per area $\rho_2\delta$ remains finite. To lowest order in $k\delta$ the growth rate of perturbations in this model is

$$\frac{\nu^2}{gk} \approx (\frac{\rho_1 - \rho_3}{\rho_1 + \rho_3})\,[1 + \frac{4k\delta\rho_1\rho_2\rho_3}{(\rho_1 + \rho_3)(\rho_1 - \rho_3)^2}],$$

$$k\delta \ll 1,\ \rho_1 \neq \rho_3 \tag{8}$$

This result indicates that an intermediate fluid layer has negligible effect on the stability of Rayleigh-Taylor modes of wavelength much longer than the layer is wide. Since the high-density layers can be much thinner than the shell as a whole, we conclude that

for important Rayleigh-Taylor modes the high-density thin shells have negligible effect on stability. Moreover, if the ablator has greater density than the fuel, ($\rho_3 > \rho_1$), then there is no interface instability.

IV. The Perturbation in the Ablation

If the adverse density gradient giving rise to the Rayleigh-Taylor instability does not occur at a contact discontinuity, but occurs rather more gradually over some finite distance, as at the ablation front, then an important feature in addition to the growth rate of the instability is the spatial location of the perturbation. In a laser-induced implosion of a pellet, it is obviously more desirable to have the instability peak farther from the ablation surface where convection,[7] firepolishing,[7] compressibility, viscosity,[3] and heat conduction[8] exert greater stabilizing effects, than closer to the pellet surface, where it is capable of causing greater damage. The nonlinear growth of a perturbation is more likely to break an imploding shell if the peak growth in the ablation occurs closer to the shell surface.

In this section we consider the simplest model of the ablation of an implosion in order to reach general conclusions regarding the location of the instability. The problem has been treated elsewhere,[4] but we reexamine it here for its significance to the question of stability of laser-fusion pellets. We will identify three distinct stabilization mechanisms associated with the density gradient of the ablation.

Our model consists of an incompressible, inviscid fluid with an exponential density gradient opposed to the effective gravity, so that the configuration is unstable. The fluid is contained between a free surface at $x = 0$, where the density is a maximum, and a rigid surface at $x = d$. The configuration is shown in Fig. 1. There is no convection or motion of the fluid in the unperturbed state. With

a density profile $\rho \propto \exp(-x/h)$, where h is a constant scale height, the differential equation (2) becomes

$$\left[\frac{d^2}{dx^2} - \frac{1}{h}\frac{d}{dx} - k^2\left(1 - \frac{g}{h\gamma^2}\right)\right] \delta u = 0 \tag{9}$$

With the free-surface boundary condition (5) applied at x = 0, and the rigid-surface boundary condition, $\delta u = 0$, applied at x = d, the growth rate is found implicitly as the positive roots of the transcendental equation

$$\tanh(\varphi_n kd) = \varphi_n kh[k^2h^2(\varphi_n^2 - 1) + \tfrac{1}{4}]^{-1} \tag{10}$$

in which $\varphi_n \equiv [1 - (g/h\gamma_n^2) + (2kh)^{-2}]^{\frac{1}{2}}$. A discrete spectrum of growth rates γ_n corresponds to the real and imaginary roots φ_n of Eq. (10). This much of the problem has been treated in Ref. 4.

This problem is just a standard Sturm-Liouville eigenvalue problem typically encountered in quantum mechanics. Here gk/γ_n^2 plays the role of the eigenvalue. The eigenfunction corresponding to the eigenvalue gk/γ_n^2 is

$$\delta u_n = \frac{\sinh \varphi_n k(d-x)}{\sinh \varphi_n kd} \exp(x/2h) \tag{11}$$

If φ_n is imaginary, as it is for all $k \geq 1/2h$ and all excited-state eigenfunctions, then (11) can be written

$$\delta u_n = \frac{\sin \mu_n k(d-x)}{\sin \mu_n kd} \exp(x/2h)$$

in which $\mu_n = i\varphi_n$.

Since the growth rate is given by

$$\frac{\gamma_n^2}{gk} = \frac{4kh}{1 + (2kh)^2(1-\varphi_n^2)} \quad ,$$

the ground-state eigenvalue (highest growth rate) corresponds to the highest allowed value of φ_n^2. Maximum growth rates are plotted versus slab width for several Rayleigh-Taylor modes in Fig. 2. If the wavenumber $k \geq 1/2h$, then the maximum growth rate is less than $(kg)^{\frac{1}{2}}$ for all slab widths, and is given by

$$\frac{\gamma_{max}^2}{kg} = \frac{4kh}{(2kh)^2 + 1} \qquad (kh \geq \tfrac{1}{2}) \tag{12}$$

Ground-state eigenfunctions, δu, for several Rayleigh-Taylor modes on a given density configuration are shown in Fig. 1. The ground state eigenfunctions have no nodes, but excited state wavefunctions oscillate sinusoidally within exponentially growing envelopes. That the eigenfunctions peak away from either boundary is a consequence of the boundary conditions, and will be the case for almost any boundary conditions. In the limit that the slab width becomes very large, the solutions become insensitive to boundary conditions, and the eigenfunctions peak towards the low density end of the configuration.

Three distinct stabilization mechanisms can be identified in Figs. 1 and 2. First, we have seen in (12) that perturbation wavelengths less than the scale height have reduced growth rates because they "sense" only a limited region of adverse density gradient. This effect is illustrated in the lowest curve in Fig. 2 for which $kh = 1$. Second, a pronounced stabilizing effect for all wavelengths is also seen in Fig. 2. The growth rate is reduced significantly from its maximum value if the slab width is less than a few scale heights. This is a thin-layer effect like that discussed in Sections II and III.

Finally, for any continuous density profile, the perturbation cannot peak at the maximum in density, as can be seen from Eq. (2), and as is illustrated in Fig. 1. In fact, it can only peak where the adverse logarithmic density gradient is sufficiently steep that $-d\ell n\rho/dx > \gamma^2/g$, since that is the condition for $d^2\delta u/dx^2 < 0$ and $d\delta u/dx = 0$. This suggests not only that shorter wavelengths are stabilized in an actual implosion, but that the perturbation is greatest somewhere outside the surface of maximum density. Irrespective of boundary conditions, the perturbation is most likely strongest near the maximum of $- d\ell n\rho/dx$. The farther from the pellet surface the perturbation peaks, the greater will be the influence of stabilization mechanisms such as viscosity and convection.

We note, however, that a more complete analysis is necessary to determine the full extent of the effects of such stabilizing mechanisms. Including these effects will alter not only the growth rates but the spatial character of the perturbations as well. Particularly in its nonlinear growth, a perturbation alters its medium and the boundaries of its medium, thereby altering the modes of its growth. Nonlinear growth may cause bubbles in the density to form and break off.[9] This process might eliminate certain modes and create new ones.

V. Conclusions

We have discussed in this paper several effects that tend to reduce the severity of the Rayleigh-Taylor instability for thin, inhomogeneous, incompressible, and inviscid laser targets. Some of these mechanisms have been treated elsewhere,[4] and are reviewed here for their relevance to the possibility of imploding thin shells stably. The effects noted here are probably insufficient by themselves to permit stable implosion of very high aspect ratio shells.

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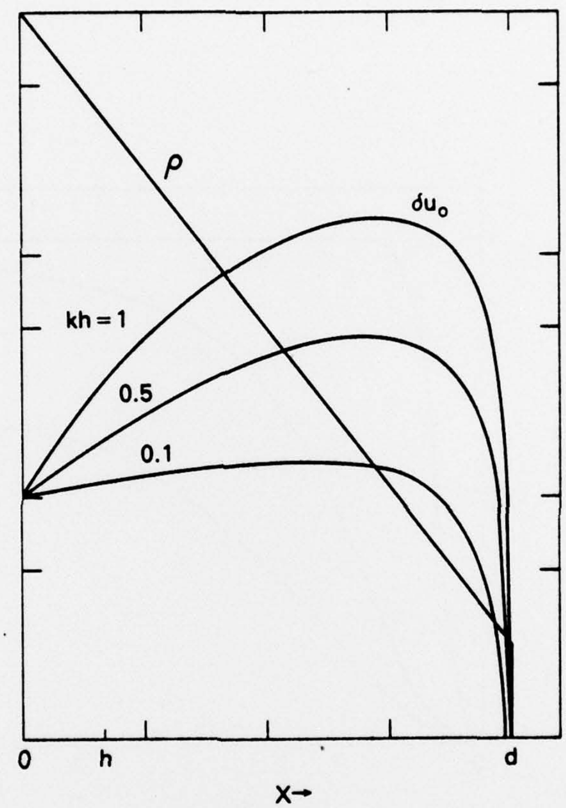

Fig. 1 — Straight line represents exponential density profile in arbitrary units with scale length h and width d = 6h. Curves are longitudinal velocity perturbation eigenmodes δu_o in arbitrary units for the three wavenumbers indicated, assuming free surface (constant pressure) at x = 0, and rigid surface at x = d. Eigenmodes are ground-state, having maximum growth rates.

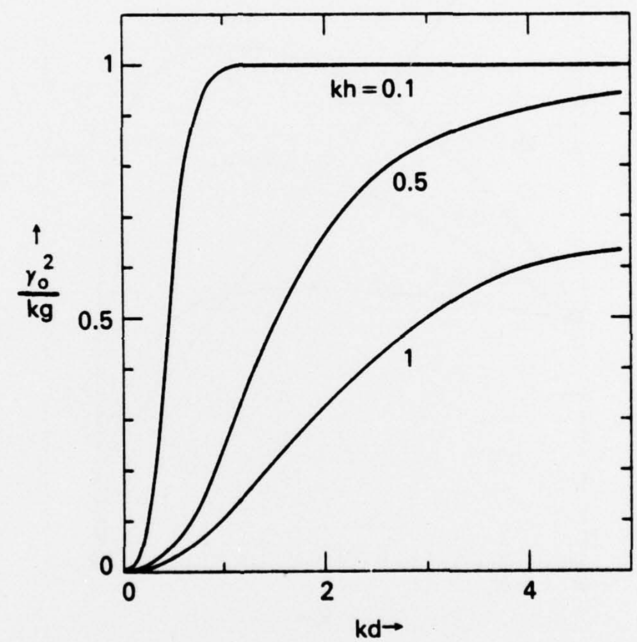

Fig. 2 — Square of Rayleigh-Taylor growth rate in units kg versus slab thickness in units 1/k for the density configuration of Fig. 1 and the wavenumbers indicated. Growth rates shown are the maximum for each wavenumber, corresponding to the ground-state eigenmodes.

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